Minimal conductivity in bilayer graphene

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Abstract. Using the Landauer formula approach, it is proven that minimal conductivity of order e^2/h found experimentally in bilayer graphene is an intrinsic property. For the case of ideal crystals, the conductivity turns out to be equal to $e^2/2h$ per valley per spin. A zero-temperature shot noise in bilayer graphene is considered and the Fano factor is calculated. Its value $1–2/\pi$ is close to the value $1/3$ found earlier for single-layer graphene.

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It has been observed recently that bilayer graphene, i.e. a two-dimensional allotrope of carbon consisting of two graphite atomic sheets, has a minimal conductivity of order e^2/h [1]. The same property has been found earlier in the single-layer graphene [2,3]. Both single- and bilayer graphene are gapless semiconductors, with conical and parabolic touching of electron and hole bands, respectively [1–3]. The charge carriers in single-layer graphene are massless Dirac fermions which is a crucial point when explaining the conductivity minimum [4–6]. Actually, this anomalous property of the two-dimensional massless fermions was considered theoretically [7–9] before the discovery of graphene. A crucial physical phenomenon here is the Zitterbewegung of quantum ultrarelativistic particles [4] which plays a role of "intrinsic" disorder; this is confirmed by calculations of the shot noise in *ideal* graphene for zero doping which turns out to have the same value (Fano factor $1/3$) as *disordered* metals [5]. At the same time, observation of the finite minimal conductivity in bilayer graphene is a serious challenge for theory [1]. Here I present a solution of this problem based on the same Landauer formula approach which was used earlier for the single-layer case [4,5].

Bilayer graphene is a zero-gap semiconductor with *parabolic* touching of the electron and hole bands described by the single-particle Hamiltonian [1,10]

$$H = \begin{pmatrix} 0 & -\left(p_x - ip_y\right)^2/2m \\ -\left(p_x + ip_y\right)^2/2m & 0 \end{pmatrix} \quad (1)$$

where $p_i = -i\hbar\partial/\partial x_i$ are electron momenta operators and m is the effective mass (here we ignore some complications due to large-scale hopping processes which are important for a very narrow range of the Fermi energies [10]).
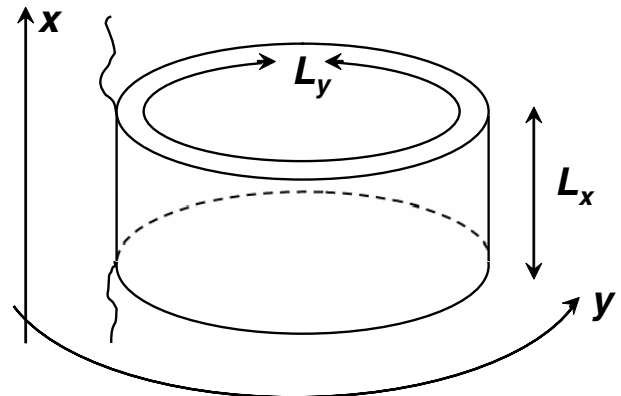
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Fig. 1. Geometry of the sample.

Two components of the wave function originate from the crystallographic structure of graphite sheets with two carbon atoms in the sheet per elementary cell. There are two touching points per Brillouin zone, K and K'. For ideal crystals, no Umklapp processes between these points are allowed and thus they can be considered independently. Our final result for the conductivity should be just multiplied by four due to two touching points and two spin projections (we will not take into account electron spin explicitly in our consideration). To calculate the conductivity at zero energy we will use the Landauer formula expressing the conductance of the system in terms of transmission coefficients. Similar to reference [4] we will use the simplest boundary conditions assuming that the sample is a ring of length L_y in the y-direction and leads connected with the sample at $x = 0$ and $x = L_x$ are made from doped bilayer graphene with potential $V_0 > 0$ and Fermi energy $E_F = -V_0 = -\hbar^2 k_F^2/2m$ (Fig. 1).

Let us first find the solution of the Schrödinger equation with zero energy, $H\Psi = 0$ where Ψ is a "spinor" with components ψ_1 and ψ_2. They satisfy the equations

$$\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)^2 \psi_2 = 0, \tag{2}$$

$$\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)^2 \psi_1 = 0. \tag{3}$$

Due to the periodicity in the y direction both wave functions should be proportional to $\exp(ik_y y)$ where $k_y = 2\pi n/L_y, n = 0, \pm 1, \pm 2, ...$ This immediately gives us the following x-dependence for the wave functions:

$$\psi_1(x) = (A_1 x + B_1) e^{k_y x},$$
$$\psi_2(x) = (A_2 x + B_2) e^{-k_y x} \tag{4}$$

$(0 < x < L_x)$. The constants A_i and B_i should be found from the boundary conditions at $x = 0$ and $x = L_x$ which are nothing but continuity conditions for both functions ψ_1 and ψ_2 and their derivatives [11].

It will be shown later that the values of k_y essential for the electron transmission are of the order of L_y^{-1} and thus much smaller than k_F in the leads. Therefore, we can restrict ourselves to the case of normal incidence only for the wave functions outside the sample:

$$\psi_1(x) = e^{ik_F x} + re^{-ik_F x} + ce^{k_F x}$$
$$\psi_2(x) = e^{ik_F x} + re^{-ik_F x} - ce^{k_F x}, \tag{5}$$

for $x < 0$ and

$$\psi_1(x) = te^{ik_F(x-L_x)} + de^{-k_F(x-L_x)}$$
$$\psi_2(x) = te^{ik_F(x-L_x)} - de^{-k_F(x-L_x)}, \tag{6}$$

for $x > L_x$. Here r and t are reflection and transmission coefficients, respectively. One should stress that to satisfy all the boundary conditions for the case of a bilayer, one has to include not only oscillatory but also exponentially decaying solutions of the Schrödinger equation [11].

Using the boundary conditions at the sample-lead boundary, one finds the set of linear equations

$$1 + r + c = B_1$$
$$1 + r - c = B_2$$
$$k_F[i(1-r) + c] = A_1 + B_1 k_y$$
$$k_F[i(1-r) - c] = A_2 - B_2 k_y$$
$$F_1 X = t + d$$
$$F_2 X^{-1} = t - d$$
$$(F_1 k_y + A_1) X = k(it - d)$$
$$(-F_2 k_y + A_2) X^{-1} = k(it + d), \tag{7}$$

where $F_i = A_i L_x + B_i, X = \exp(k_y L_x)$.

By use of the assumptions

$$k_F \gg k_y, L_x^{-1} \tag{8}$$

one can easily solve the equations (7) and find

$$t = \frac{2iL_x}{k_F} \frac{\cosh(k_y L_x)}{L_x^2 + \frac{2i}{k_F^2}\cosh^2(k_y L_x)}. \tag{9}$$

Corrections to this formula are of order of $1/(k_F L_x)$; we cannot keep them in the answer since terms of the same order of magnitude have been omitted by considering the normal-incidence case for the wave functions in the leads (5), (6).

Thus, for the transmission coefficient $T_n = |t(k_y = 2\pi n/L_y)|^2$ one obtains the final result

$$T_n = \frac{4k_F^2 L_x^2 \cosh^2(k_y L_x)}{k_F^4 L_x^4 + 4\cosh^4(k_y L_x)}. \tag{10}$$

One can see that the transmission coefficient reaches a maximum value equal to 1 at $\cosh(k_y L_x) = k_F L_x/\sqrt{2}$ or, approximately, at

$$k_y/k_F \simeq \ln\left(\sqrt{2}k_F L_x\right)/(k_F L_x) \tag{11}$$

which obviously satisfies the condition (8) for macroscopically large $L_x \gg k_F^{-1}$. Note that complete transmission through the potential barrier for some finite incident angles is a characteristic property of the bilayer case, in contrast with the single layer, where complete transmission takes place at exactly normal incidence [11].

Using the Landauer formula (for review, see Refs. [12,13]) one can calculate the conductance per valley per spin

$$g = \frac{e^2}{h}\sum_{n=-\infty}^{\infty} T_n. \tag{12}$$

Similar to references [4,5] to calculate the conductivity of bilayer graphene at zero energy one should consider the case $L_y \gg L_x$. In that case the sum in equation (12) can be replaced by an integral. Introducing the integration variable $z = \cosh(2k_y L_x) + 1$ and taking into account condition (8) one finds for the conductivity $\sigma = (L_x/L_y) g$:

$$\sigma = \frac{e^2}{2h}. \tag{13}$$

Thus, the conductivity of bilayer graphene has the same order of magnitude as for the single-layer case (where the coefficient $1/\pi$, instead of $1/2$, was obtained by similar method in Refs. [4,5]). This result looks rather unexpected since the electron spectra in these two cases are drastically different. More accurate calculations of the integral gives a correcting multiplier $1 + \frac{4\ln(k_F L_x)}{\pi k_F^2 L_x^2} + ...$ in equation (13).

Following reference [5] one can estimate the Fano factor characterizing the intensity of electron shot noise:

$$F = \frac{\sum\limits_{n=-\infty}^{\infty} T_n(1 - T_n)}{\sum\limits_{n=-\infty}^{\infty} T_n} \tag{14}$$

(for a general review of quantum-limited shot noise and physical meaning of the Fano factor, see Refs. [13,14]). A straightforward calculation for the case $L_y \gg L_x$ gives us the answer

$$F = 1 - \frac{2}{\pi} \qquad (15)$$

which is rather close to the value $1/3$ found for the case of the single layer, as well as for the case of disordered metals [5]. This means that, in a sense, the case of bilayer graphene is also characterized by some "intrinsic" disorder similar to the Zitterbewegung [4].

Unfortunately, the accuracy of experimental data [1] is not sufficient to establish the numerical coefficient in the expression for the minimal conductivity. For the case of a single layer it is close to 1, instead of $1/\pi$.

To conclude, we have demonstrated that the Dirac energy spectrum is actually not important for the existence of minimal conductivity in graphene. The latter has the same order of magnitude both for conical (a single-layer case) and for parabolic (a bilayer case) energy spectrum near a band crossing point.

References

1. K.S. Novoselov, E. McCann, S.V. Morozov, V.I. Falko, M.I. Katsnelson, U. Zeitler, D. Jiang, F. Schedin, A.K. Geim, Nature Phys. **2**, 177 (2006)
2. K.S. Novoselov, A.K. Geim, S.V. Morozov, D. Jiang, M.I. Katsnelson, I.V. Grigorieva, S.V. Dubonos, A.A. Firsov, Nature **438**, 197 (2005)
3. Y. Zhang, Y.-W. Tan, H.L. Stormer, P. Kim, Nature **438**, 201 (2005)
4. M.I. Katsnelson, Eur. Phys. J. B **51**, 157 (2006)
5. J. Tworzydlo, B. Trauzettel, M. Titov, A. Rycerz, C.W.J. Beenakker, Phy. Rev. Lett. **96**, 246802 (2006)
6. K. Ziegler, e-print `arXiv:cond-mat/0604537`
7. E. Fradkin, Phys. Rev. B **33**, 3263 (1986)
8. P.A. Lee, Phys. Rev. Lett. **71**, 1887 (1993)
9. A.W.W. Ludwig, M.P.A. Fisher, R. Shankar, G. Grinstein, Phys. Rev. B **50**, 7526 (1994)
10. E. McCann, V.I. Falko, Phys. Rev. Lett. **96**, 086805 (2006)
11. M.I. Katsnelson, K.S. Novoselov, A.K. Geim, e-print `arXiv:cond-mat/0604323`
12. C.W.J. Beenakker, H. van Houten, Solid State Phys. **44**, 1 (1991)
13. Ya. M. Blanter, M. Büttiker, Phys. Rep. **336**, 1 (2000)
14. C. Beenakker, C. Schönenberger, Phys. Today, May, 37 (2003)

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